V ASSIGNMENT- IVCS/IT

1. Show that ‾X, the sample mean is both an unbiased and consistent estimator for the population mean.
2. Let X have a pdf of the form

f(x;θ) = θxθ-1, 0 < x< 1,

= zero elsewhere, where θ∈ { θ : θ = 1, 2 }.

To test the simple hypothesis H0 : θ = 1 against the alternative simple hypothesis H0 : θ = 2, use a random sample X1, X2 of size n = 2 and define the critical region to be . Find the power function of the test.

1. Let (X1, X2 ,………Xn) denote a random sample of size n from the distribution with pdf 

Find MLE for θ.

1. Let (X1, X2, …, Xn) denote a random sample from a distribution which is  . Find maximum likelihood estimators for .
2. Let  and S2 be the mean and variance of a random sample of size 25 from a N (3, 100). Evaluate Pr{ 0 < <6, 55.2 < S2 < 145.6}.
3. Let a random sample of size 17 from a normal distribution yield =4.7, S2 = 5.76. Determine a 90% confidence interval for .
4. Let ‾X be the mean of a random sample of size n from distribution which is N(3, 9). Find n such = 0.90, approximately.
5. Let us assume that the life length of a tyre in miles, say X is normally distributed with mean θ and standard deviation 5000. Past experience indicates that θ = 30,00 the manufacturer claims that the tyres made by a new procedure have mean θ > 30,000 and it is very possible that θ = 35, 000. Let us check this claim by testing H0 : θ < 30,000 against H1 : θ > 30,000. We shall observe n independent values of X say X1, X2, …, Xn and we shall reject H0 if and only if ‾x ≥ c . Determine n and c so that the power function K(θ) of the test has values K (30,000) = 0.01 and K (35,000) = 0.98.
6. Let (X1, X2, …Xn) be a random sample of size n form a distribution N(θ, 100). Show that  is a best critical region for testing H0 : θ = 75 against H1 : θ = 78. Find n and c so that  approximately.
7. Compute an approximate probability that mean of a random sample of size 15 from a distribution having pdf 
8. The Mendelian theory states that the probabilities of classification a, b, c, d are respectively . From a sample of 160 the actual numbers observed were 86, 35, 26 and 13. Is this data consistent with the theory at 0.01 significance level.
9. Let X have binomial distribution with parameters n = 10 and p. The simple hypothesis H0 : p=  is rejected and the alternative simple hypothesis H1 : p=  is accepted, if the observed value of X1 , a random sample of size 1, is less than that or equal to 3. Find the power function of the test.
10. A survey of 320 families with 5 children each, revealed the following distribution. Is the result consistent with the hypothesis that male and female births are equally probable at 0.01 significance level.

No. of Boys 5 4 3 2 1 0

No. of Girls 0 1 2 3 4 5

No. of families 14 56 110 88 40 12

1. Let (X1, X2) be a sample of size 2 from the distribution having the pdf

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We reject H0 : θ =2 and accept H1: θ =1, if the observed values (x1, x2) are such that .

Find significance level of the test and the power of the test when H0 is false.

1. A die is cast n = 120 independent times and the following resulted.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Spots up | 1 | 2 | 3 | 4 | 5 | 6 |
| Frequency | b | 20 | 20 | 20 | 20 | 1. – b |

If we use chi-square test, for what values of b would the hypothesis that the die is unbiased be rejected at 0.025 significance level.

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